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Technical Comments

Comment on "Effect of Thrust/Speed Dependence on Long-Period Dynamics in Supersonic Flight"

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Introduction

REFERENCE 1 discusses effects of speed-dependent variations in thrust on the long-period motions of aircraft in supersonic flight. Its principal contribution is the demonstration of effects that a complex pair of zeros have on speed-to-throttle feedback. It is concluded that there are fundamental differences between thrust/speed effects in subsonic and supersonic flight. It also is stated that "thrust variations with speed (or Mach number) have practically no effect on the phugoid in supersonic flight," then that "the phugoid is not influenced at all in supersonic flight."[1] The purpose of this Comment is to show that thrust/speed effects are indeed small but not zero in supersonic flight and that this is a kinematic effect of increasing speed rather than an aerodynamic effect of Mach number. In the process, a simplified analytical model is offered for further study. Furthermore, it is noted that pitching moment/speed sensitivity due to thrust-axis offset is likely to have important long-period effect even when the direct thrust/speed effects are small.

Background

Small-scale longitudinal motions of aircraft typically are described by a fourth-order set of ordinary, linear, time-invariant differential equations, derived under the assumption of constant gravitational acceleration and air density. As long as round-Earth effects are negligible, these equations apply equally well to aircraft flying at both subsonic and supersonic speeds. Two modes of oscillatory motion are evidenced by these equations: the long-period interchange of point-mass kinetic and potential energy christened the "phugoid mode" by Lanchester in 1897,[2] and the short-period attitude motions about the center of mass. Under certain circumstances, such as low-speed flight, strong pitch sensitivity to airspeed, lack of inherent pitch stability, or abrupt control input, translational and rotational motions may evidence significant coupling. Otherwise, these motions are largely uncoupled.

In 1942, Scheubel noted the effect of air-density gradient (with altitude) on limiting the phugoid period.[3] In 1950, Neumark identified a third mode of motion (the "height mode") arising from atmospheric gradients and considered its effect on supersonic flight.[4] Although the height mode is principally due to the physics of the atmosphere, its characteristics are linked parametrically to the phugoid mode, as described below. Etkin drew the connection between the height mode and orbital motions in 1960,[5] and the present author provided a comprehensive study of atmospheric-gradient, thrust-law, and pitch-dynamic effects on supersonic flight in 1969.[6] The dynamic equations of Ref. 1 are essentially those of Ref. 6, expressed with nondimensionalizing time and mass parameters and neglecting pitch/speed-stability and pitch/altitude effects. Sachs considered pitching effects on long-period supersonic motions in 1972.[7] More recently, the combined effects of thrust law, round-Earth terms, and atmospheric gradients on hypersonic flight were addressed by Markpoulos et al. in Refs. 8 and 9, while Berry has noted the likelihood that thrust/speed sensitivities and long-period root locations of hypersonic configurations may change markedly with Mach number.[10]

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Technical Discussion

Reference 1 implies that the mechanisms for phugoid thrust/speed dependence are different in subsonic and supersonic flight, but the difference is kinematic not aerodynamic, as the form of the equations is unchanged by airspeed or Mach number (although values of the aerodynamic coefficients and derivatives may change). Similarly, the thrust feedback analysis presented in Ref. 1 is structurally the same for both subsonic and supersonic flight. The open-loop thrust/speed sensitivity called n_u in Ref. 1 and ν' in Ref. 6 has a direct effect on phugoid damping and virtually no effect on phugoid period in both flight regimes. This is shown for a subsonic case in Fig. 1 (Fig. 1 of Ref. 1) and for a supersonic case in Fig. 2 (Fig. 4 of Ref. 6). The latter figure presents results for a hypothetical $M = 3$ supersonic transport.

As noted in Ref. 6, the thrust/speed sensitivity reflects the exponent of thrust/speed dependence; for the thrust model $T = T_0 + T_1 V^{\nu}$, where V is the airspeed, $T_{0,1}$ are model constants, and ν' is $\nu[1 - (T_0/T)]$. The ν' range considered there (-1 to 3) encompasses characteristics of the airbreathing engines likely to be used in supersonic flight. Figure 2 illustrates that ν' has negligible effect on the phugoid period, that the phugoid mode is nearly neutrally stable in all supersonic cases, that the effect of ν' on phugoid damping ratio ζ_p is small but linear, and that ν' has an inverse effect on height-mode time constant τ_h. This is consistent with the result of Fig. 1, which is plotted for an n_u (or ν') range of 0 to -10. (The gain through which airspeed is fed back to thrust need not be limited to the natural range of airbreathing-engine open-loop response.) The corresponding supersonic root locus would be near the imaginary ($i\omega$) axis for ν' in $(-1,3)$, where damping ratio is changing but natural frequency is not.

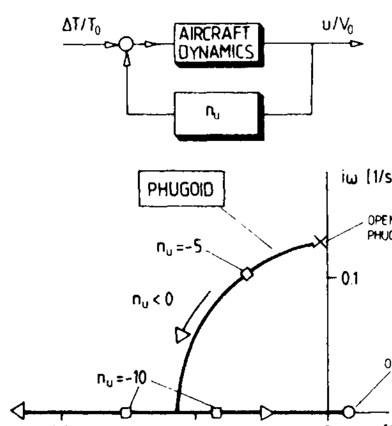

Fig. 1 Effect of speed feedback to thrust on long-term dynamics in subsonic flight ($M = 0.25$) (from Ref. 1).

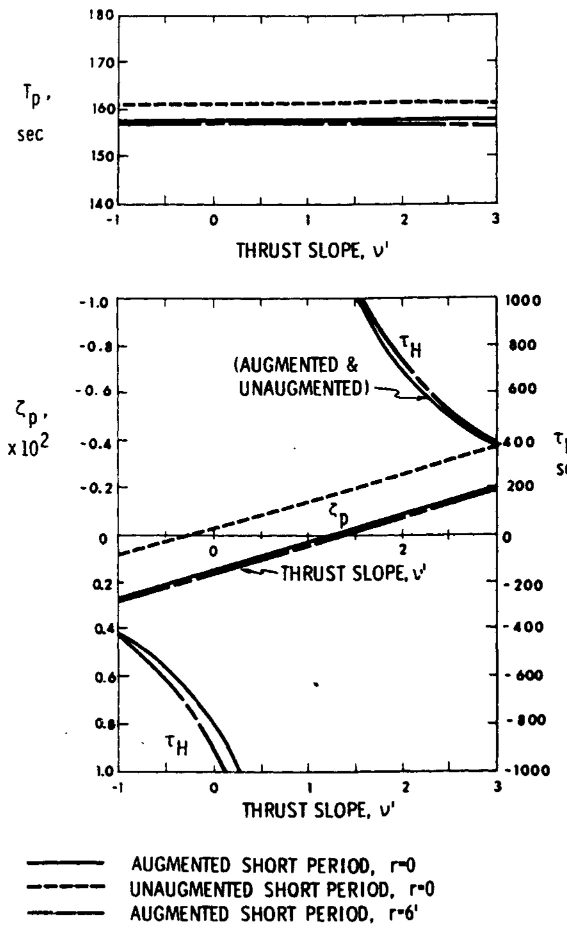

Fig. 2 Thrust-slope effect on the phugoid and height modes (from Ref. 6).

Table 1 of Ref. 6 presents qualitative results of feeding back motion variables to thrust, lift, and moment controls in supersonic flight, based on examination of the corresponding transfer functions. The table indicates that forward velocity feedback to thrust stabilizes both height and phugoid modes, although the latter effect is limited. Reference 1 reveals the reason: a pair of zeros in close proximity to the phugoid poles. Table 1 (Ref. 6) implies that the double-loop closure investigated in Ref. 1 (pitch angle to moment and airspeed to thrust) should perform well, and an alternate double-loop closure (airspeed to moment and altitude to thrust) is demonstrated to be satisfactory in Ref. 6. Berry suggests that the second double-loop closure is preferable to the first for controlling the long-period dynamics of hypersonic vehicles like the National Aerospace Plane.[10]

Simplified Analysis

To better understand thrust/speed effects on long-period motions in the absence of significant pitching moment, a third-order flat-Earth model can be examined. For an aircraft in straight-and-level flight at a nominal airspeed of V_n,

$$\begin{bmatrix} \Delta \dot{V} \\ \Delta \dot{\gamma} \\ \Delta \dot{z} \end{bmatrix} = \begin{bmatrix} TD_V & -g & TD_z \\ L_V/V_n & 0 & L_z/V_n \\ 0 & -V_n & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \gamma \\ \Delta z \end{bmatrix} + \begin{bmatrix} T_{\delta T} \\ 0 \\ 0 \end{bmatrix} \delta T \quad (1)$$

ΔV, $\Delta\gamma$, and Δz are the perturbations in airspeed, flight-path angle, and altitude (positive down), $(\dot{\ })$ represents differentiation with respect to time, and δT is the throttle setting. The matrix elements are dimensional stability-and-control derivatives expressing the sensitivity of specific forces to variations in airspeed, altitude, and throttle setting. Assuming that the aircraft is flying in an isothermal layer (whose byproduct is a constant speed of sound), that thrust is proportional to air density, that thrust and drag are nominally balanced, and that the nominal lift equals weight, the stability derivatives are

$$TD_V = [\nu' - V_n(C_{D_V}/C_{D_n}) - 2]D_n/mV_n \quad (2)$$

$$TD_z \approx 0 \quad (3)$$

$$L_V = [(C_{L_V}/C_{L_n}) + (2/V_n)]g \quad (4)$$

$$L_z = g\rho_z/\rho_n \quad (5)$$

where D_n, C_{D_n}, C_{L_n}, and ρ_n are the nominal values of drag force, drag coefficient, lift coefficient, and air density. C_{L_V} and C_{D_V} are the sensitivities of lift and drag coefficients to airspeed, whereas m, g, and ρ_z represent aircraft mass, gravitational constant, and air density change with altitude. TD_V contains the thrust/speed dependence, as well as the drag/speed dependence, for the only effect of concern is the difference between the two. Thrust is assumed to be aligned with the velocity vector; therefore, throttle settings have direct effect on $\Delta\dot{V}$ only, and $T_{\delta T}$ is $\partial(T/m)/\partial\delta T$. Calling the stability matrix F, the characteristic polynomial $\Delta(s)$ is the determinant of $(sI - F)$, and the characteristic equation is

$$\Delta(s) = s^3 - TD_V s^2 + [(gL_V/V_n) + L_z]s + (L_V TD_z - L_z TD_V) = 0 \quad (6)$$

Assuming that the phugoid mode is several times faster than the height mode (as is usually the case), the Bairstow approximation can be used to estimate the roots.[11] The first three terms approximate phugoid dynamics, whereas the last two approximate height-mode motion. Hence, the phugoid natural frequency ω_{n_p} and damping ratio ζ_p are

$$\omega_{n_p} \approx \sqrt{(gL_V/V_n) + L_z} \quad (7)$$

$$\zeta_p \approx -TD_V/2\omega_{n_p} \quad (8)$$

whereas the height-mode time constant τ_z (called τ_h earlier) is

$$\tau_z \approx -\frac{(gL_V/V_n) + L_z}{L_V TD_z - L_z TD_V} \quad (9)$$

Equations (4) and (5) confirm the trends of Fig. 2, which were computed for the higher-order dynamic model. Also note that τ_z is proportional to $\omega_{n_p}^2$, with its sign being determined by the difference of the denominator terms in Eq. (9) (normally positive (stable), given Eq. (3). As the second term of the characteristic polynomial $(-TD_V)$ represents the "total damping" of the dynamic system, the thrust/speed dependence determines how positive or negative the sum of phugoid and height-mode roots is, but the other parameters of the equation determine how well-damped each mode is. For fixed TD_V, parametric variations that increase phugoid damping decrease height-mode damping, and vice versa.

The zeros of speed-to-throttle feedback in this simplified model are the roots of

$$s^2 + L_z = 0 \quad (10)$$

L_z is positive, and the roots of Eq. (6) are imaginary, being located at $\pm i\sqrt{L_z}$. Thus, the zeros are smaller than the phugoid poles (as L_V also is positive), and they lie on the imaginary axis, in general agreement with Ref. 1. Equations (1-10) apply equally well in both subsonic and supersonic flight. L_V/V_n decreases as V_n increases, whereas L_z remains constant; hence, the phugoid roots approach the zeros, causing the thrust/speed dependence to have less and less effect on phugoid root variations.

References 6 and 7 both illustrate that pitching moment sensitivity to speed and altitude variations arising either from aeroelastic effects or thrust-axis offset can have a larger influence on the characteristics of supersonic long-period motions than thrust/speed effects alone, including large changes in ω_{n_p} and τ_z, as well as negative (unstable) values of ζ_p. This moment-producing characteristic provides additional reason to consider the speed-to-moment/altitude-to-thrust controller. Short-period stability augmentation may have a beneficial effect on long-period stability, although overall stability can be assured only by coordinated design of the full-order control system.

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Reply by Author to Robert F. Stengel

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THE preceding Technical Comment of Stengel[1] provides valuable additional information about thrust/speed and pitching moment/speed effects and helps to clarify supersonic flight dynamics characteristics that show significant differences when compared with well-known relationships in subsonic flight. More information and further insight into the problems of supersonic flight are provided by a comprehensive paper of the same author in Ref. 2 that includes the effects of control-loop closures and related favorable control laws and the effects of atmospheric disturbances. In recent papers,[3-5] additional topics concerning the effect of thrust characteristics and their significance for supersonic and hypersonic flight are considered.

The central issue of Ref. 6 concerns the direct thrust/speed effects on long-period dynamics in supersonic flight and their differences as regards well-known characteristics attributed to these effects. Particular emphasis is given to the phugoid and its insensitivity concerning the effect of thrust changes with speed. This effect can be qualified as negligible in supersonic flight or (practically) not existent. It pronouncedly contrasts with the well-known subsonic characteristics according to which the phugoid is substantially influenced by the thrust variations with speed. It is the purpose of this reply to provide a more detailed insight which helps to point out more clearly the central issue of Ref. 6.

In Ref. 6, the root locus technique was applied for showing the effect of thrust changes with speed. From the results presented, it follows that the maximum change in phugoid damping ratio for the whole root locus is on the order of $\zeta_{p\max} - \zeta_{p\min} = 0.02$. This holds for an unlimited gain of the root locus, i.e., for hypothetical thrust changes of unlimited size. Compared to such large thrust changes, a maximum change in damping ratio $\zeta_{p\max} - \zeta_{p\min} = 0.02$ may be qualified as negligible. This contrasts with the substantial effect of thrust/speed dependence in subsonic flight where the change in phugoid damping ratio reaches the maximum value possible $\zeta_{p\max} = 1$ (i.e., the phugoid is changed from an oscillation lightly damped to an aperiodic mode of motion). This already holds for moderate gains (see Fig. 1 in Ref. 6).

A simplified analysis may provide more insight into the problem addressed by considering analytical expressions for the long-period modes of motion. An adequate mathematical model for a horizontal reference flight condition in the absence of significant pitching moments may be expressed as (with the notation used in Ref. 1)

$$\begin{bmatrix} \Delta \dot{V} \\ \Delta \dot{\gamma} \\ \Delta \dot{z} \end{bmatrix} = \begin{bmatrix} TD_V & -g & TD_z \\ L_V/V_n & 0 & L_z/V_n \\ 0 & -V_n & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \gamma \\ \Delta z \end{bmatrix} + \begin{bmatrix} T_{\delta T} \\ 0 \\ 0 \end{bmatrix} \delta T \quad (1)$$

The characteristic equation related to this system is

$$\Delta(s) = s^3 + Bs^2 + Cs + D = 0 \quad (2a)$$

where

$$B = -TD_V, \quad C = \frac{gL_V}{V_n} + L_z, \quad D = L_V TD_z - L_z TD_V \quad (2b)$$

There are three roots $s_{1,2,3}$ of which one usually is real valued ($s_1 := s_h$: height mode) and the others are complex ($s_{2,3} := -\zeta_p \omega_{n_p} \pm i\omega_{n_p}\sqrt{1-\zeta_p^2}$: phugoid). As regards their relative magnitude, the following relations usually hold:

$$|s_h| \ll |\omega_{n_p}|, \qquad |\zeta_p| \ll 1 \quad (3)$$

The connection between the roots and the coefficients of the characteristic equation is given by

$$s_h - 2\zeta_p \omega_{n_p} = -B, \quad \omega_{n_p}^2 - 2s_h \zeta_p \omega_{n_p} = C, \quad s_h \omega_{n_p}^2 = -D \quad (4)$$

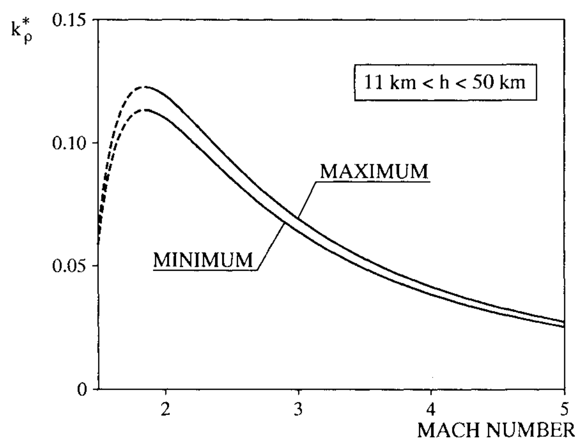

Fig. 1 Factor k_p^* as a function of Mach number (atmospheric data from Ref. 7).

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